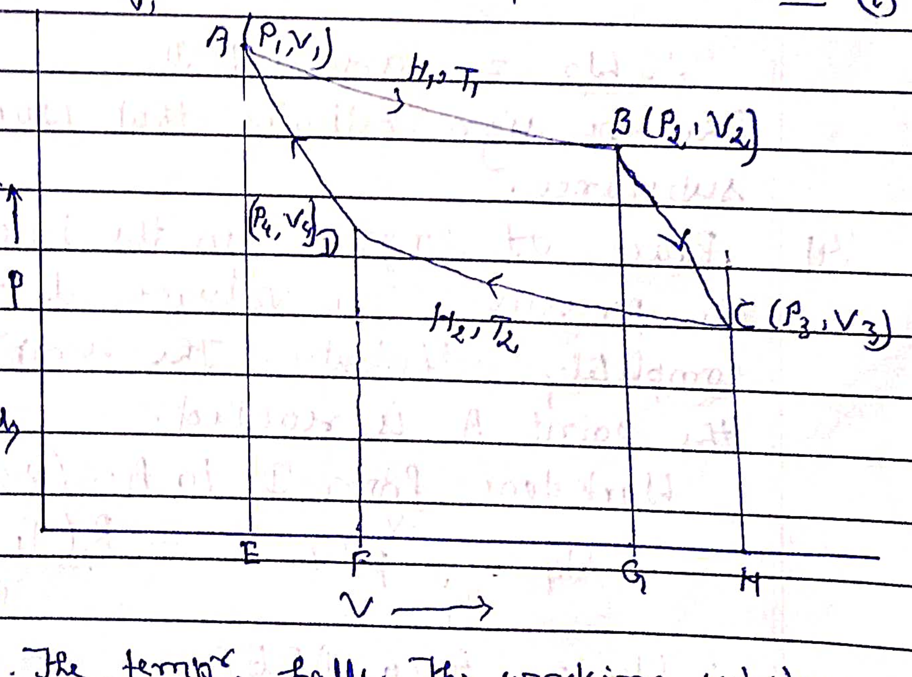
Carnot's Cycle :-

(1) Place the engine containing the working substance over the source at tempr. T_1. The working substance is also at a tempr. T_1. Its pressure is P_1 and volume is V_1 as shown by the point A. Decrease the pressure. The volume of the working substance increases. Work is done by the working substance. As the bottom is perfectly conducting to the source at tempr. T_1, it absorbs heat. The process is completely isothermal. The tempr. remains constant. Let the amount of heat absorbed by working substance be H_1 at the tempr. T_1. The point B is obtained.

Let us suppose one gram molecule of the working substance.

Workdone A to B, $W_1 = \int_{V_1}^{V_2} P.dV = RT_1 \log \frac{V_2}{V_1}$ = Area ABG_1E — (i)

(2) Place the engine on the stand having an insulated top. Decrease the pressure on the working substance. The volume increases. The process is completely adiabatic. Work is done by the working substance at the cost of



its internal energy. The tempr. falls. The working substance undergoes adiabatic change from B to C. At C, the tempr. is T_2.

Work done from B to C (adiabatic process)

$$W_2 = \int_{V_2}^{V_3} P.dV$$

$$= \int_{V_2}^{V_3} \frac{dV}{V^\gamma}$$

But PV^γ = Constant = K

$P_2V_2 = RT_1$

$P_3V_3 = RT_2$

$P_3V_3^\gamma = P_2V_2^\gamma = K$

$$\therefore W_2 = \frac{KV_3^{1-\gamma} - KV_2^{1-\gamma}}{1-\gamma} = \frac{P_3V_3 - P_2V_2}{1-\gamma}$$

$$= \frac{R[T_2 - T_1]}{1-\gamma}$$

$$= \frac{R[T_1 - T_2]}{\gamma - 1}$$

$\therefore W_2 =$ Area BCHG —— (ii)

(3) Place of engine on the sink at tempr. T_2. Increase the pressure. The work is done on the working substance. As the base is conducting to the sink, the process is isothermal. A quantity of heat H_2 is rejected to the sink at tempr. T_2. Finally the point D is reached.

Workdone from C to D (isothermal process)

$$W_3 = \int_{V_3}^{V_4} p\,dV = RT_2 \log \frac{V_4}{V_3} = -RT_2 \log \frac{V_3}{V_4}$$

$\therefore W_3 =$ area CHFD —— (iii)

The -ve sign indicates that work is done on the working substance.

(4) Place of engine on the insulating stand. Increase the pressure. The volume decreases. The process is completely adiabatic. The tempr. rises and finally the point A is reached.

Workdone from D to A. (adibatic process)

$$W_4 = \int_{V_4}^{V_1} p\,dV = -\frac{R(T_1 - T_2)}{\gamma - 1}$$

$\therefore W_4 =$ Area DFEA — (iv)

W_2 and W_4 are equal and opposite and cancel each other.

The net workdone by the working substance in one complete cycle.

$=$ Area ABGE + Area BCHG − Area CHFD − Area DFEA

$=$ Area ABCD

The net amount of heat absorbed by the working substance

$= H_1 - H_2$

Net work $= W_1 + W_2 + W_3 + W_4$

$$W = RT_1 \log \frac{V_2}{V_1} + \frac{R(T_1 - T_2)}{\gamma - 1} - RT_2 \log \frac{V_3}{V_4} - \frac{R(T_1 - T_2)}{\gamma - 1}$$

$$\therefore \quad W = RT_1 \log \frac{V_2}{V_1} - RT_2 \log \frac{V_3}{V_4} \quad \text{— (v)}$$

The points A and D are on the same adiabatic

$$T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \quad \text{— (vi)}$$

The points B and C are on the same adiabatic.

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \quad \text{— (vii)}$$

from eqn. (vi) & (vii), we get,

$$\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

from eqn (v)

$$W = RT_1 \log \frac{V_2}{V_1} - RT_2 \log \frac{V_2}{V_1}$$

$$\therefore W = R\left[\log \frac{V_2}{V_1}\right]\left[T_1 - T_2\right]$$

$$\therefore W = H_1 - H_2$$

Efficiency :— $\eta = \frac{\text{Useful Output}}{\text{Input}} = \frac{W}{H_1}$

Heat is supplied from the source from A to B only.

$$H_1 = RT_1 \log \frac{V_2}{V_1}$$

$$\therefore \eta = \frac{W}{H_1} = \frac{H_1 - H_2}{H_1}$$

$$\therefore \eta = \frac{R(T_1 - T_2)\log \frac{V_2}{V_1}}{RT_1 \log \frac{V_2}{V_1}}$$

or

$$\eta = 1 - \frac{H_2}{H_1}$$

$$\therefore \eta = 1 - \frac{T_2}{T_1} \quad \text{— (viii)}$$

The carnot's engine is perfectly reversible. It can be operated in the reverse direction also. Then it works as a refrigerator. The heat H_2 is taken from the sink and external work is done on the working substance and Heat H_1 is given to the source at the higher temperature.

The isothermal process will take place only when the piston moves very slowly to give enough time for the heat transfer to take place. The adiabatic process will take place when the piston moves extremely fast to avoid heat transfer. Any practical engine cannot satisfy these conditions.

All practical engines have an efficiency less than the carnot's engine.